Norwood's dome: a revolution in incident-light photographic exposure metering

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ABSTRACT AND INTRODUCTION

In the late 1930's, Donald W. Norwood introduced a new principle of incident light photographic exposure metering in which a translucent hemispherical shell (a "dome") collects the ambient light incident on the scene for measurement by a photoelectric cell. It was found that exposure meters following this principle could, with a single measurement, consistently develop a photographic exposure recommendation that would be highly appropriate over a range of lighting situations, especially those of interest in cinematography.

Today, the preponderance of "serious" incident light photographic exposure meters exploit Norwood's principle.

But it is not at all obvious, even after considerable study, just how and why meters following Norwood's principle give this widely-acclaimed performance. In this article, we will look "under the dome" and see just what is going on.

Background is given in various pertinent aspect of the topic of photographic exposure metering. An appendix gives an analysis and critique of Norwood's seminal paper on this system, and another gives the derivation of the theoretical directivity of a meter with a hemispherical receptor.

1. PHOTOGRAPHIC EXPOSURE METERING

1.1 The concept

In photographic exposure metering, we use a special instrument which determines either the average luminance of the scene to be photographed or the illuminance of the illumination on the scene, and from that (along with the known or assumed sensitivity of the film or digital sensor system in use) provides us with a *photographic exposure recommendation*. By that we mean a continuum of combinations of exposure time (shutter speed) and aperture (as an f-number) that would all produce the same photometric impact on the film or sensor. Our aspiration is that by using that exposure recommendation for our "shot" we will attain the desired *exposure objective*.

1.2 The exposure objective

What do we mean by *exposure objective*? The "scene" being photographed, from an exposure standpoint (and let's assume a "monochrome" camera) presents to the camera as a mosaic of varying *luminance*, with a certain overall range. The lens transforms this into a mosaic of *illuminance* upon the film or sensor.

We would like the range of illuminance in that mosaic to be "planted" so that, in combination with some exposure time (shutter speed), the resulting range of *photometric exposure*[1] will fall in an appropriate place in the acceptable range of photometric exposure of the film or sensor.

But what is "appropriate"? There are several strategies we might adopt. Two commonly-chosen ones are:

A. "Expose to the right" [2]. Here we seek to have the "brightest" spots in the scene receive photometric exposure that is "close to saturation"—that is, close to the photometric exposure above which changes in photometric exposure do not result in very much change in the response.

B. "Reflectance-based" [3]. Here we seek to map the portions of the scene having different reflectances approximately onto proportional values of photometric exposure (on a scale that runs to 100% at the "saturation" photometric exposure).

An advantage of (A) is that the range of the film or sensor is best exploited with regard to such performance properties as dynamic range and noise performance. A common metaphor for an important disadvantage of (A) is that, if we achieve it, the image of a "gray cat on an ash pile" (nothing else in the scene) will look like a "white cat on a snowdrift".

An advantage of (B) is that, following the metaphor above, the images will reveal the various objects (cats, what the cats sit on) as we expect to see them. Stuff we know to be "gray" will in the image look

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[1] *Photometric exposure* is the phenomenon to which the film or sensor responds, the product of the illuminance on the film or sensor and the exposure time.

[2] So called because "to the right" is the direction of increase in photometric exposure, exposure result, and such in various charts, histogram displays, and so forth.

[3] This very much follows the underlying concept of the Zone System, a doctrine of exposure planning devised and promoted by Ansel Adams and others.

"gray", regardless of the overall scene content; stuff we know to be "white" will look "white".

1.3 Reflected-light exposure metering

The earliest approach to exposure metering, and still widely-used, is *reflected-light metering*. Here our instrument measures the average luminance of the scene (over a certain field of view, which may or may not closely conform to the field of view of the camera as it will be used to photograph the scene). We also feed into the meter an *exposure index*, which in basic practice would be the advertised *ISO speed* of the film or digital sensor system. The instrument then gives us a photographic exposure recommendation (defined earlier).

If we actually follow that recommendation in setting the camera for our shot, the result will be that the *average* photometric exposure on the film or digital sensor will be a fixed value (with reference to the sensitivity of the film of sensor).

Which of the two often-cited exposure objectives, A and B, will this fulfill? Neither. Resorting to the “cat” metaphor used above, this metering technique results the image of a "white cat on a snowdrift" (nothing else in the scene), or a “black cat on a coal pile" (nothing else in the scene), both looking like a “gray cat on an ash heap".

Then why do we use this metering technique? Because it is easy to do.

1.4 Incident light exposure metering

Here our instrument determines what we for the moment will describe as the *illuminance* of the light that is incident on the scene. We also feed into the meter an *exposure index*, discussed above. The instrument then gives us a photographic exposure recommendation.

Ideally, if we actually follow that recommendation in setting the camera for our shot, we will attain exposure objective B. In the image, for each scene element, the relative luminance of the image will be that proportion of the maximum recordable luminance that is the reflectance of the scene element.

But in fact, if the illumination of the subject does not come uniformly from all directions, this tidy result will only be achieved if all surfaces of interest in the scene have the same orientation (are all parallel to a certain plane), and the incident illuminance is measured with respect to that plane.

That is hardly the case in most photography and cinematography. For a human subject, a small region in the center of the forehead is in a different plane than a certain small region on one cheek.

1.5 A further complication

A further complication is that we often do not really want to attain Objective B. A powerful example of why is if we photograph a spherical object of uniform surface reflectance (perhaps a decorative polished gray stone ball ball). If we perfectly attain objective B, the result will be as seen in figure 1.

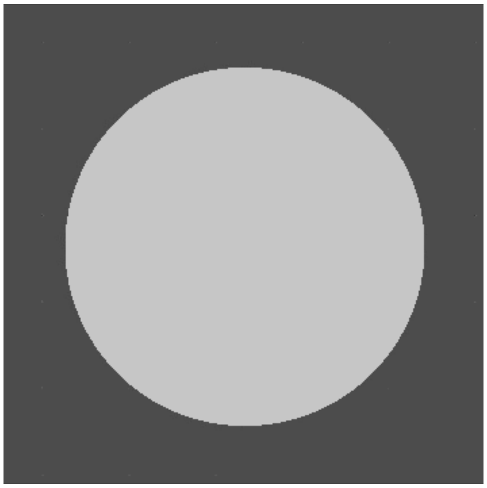

Figure 1. Sphere under uniform omnidirectional illumination

And the "implied relative illuminance" of the image of the sphere would ideally be (as part of objective B) the same as the reflectance of the sphere itself.

But, sadly, that doesn't look so much like a sphere. We might expect a photo of a sphere to look like one of the images in figure 2.

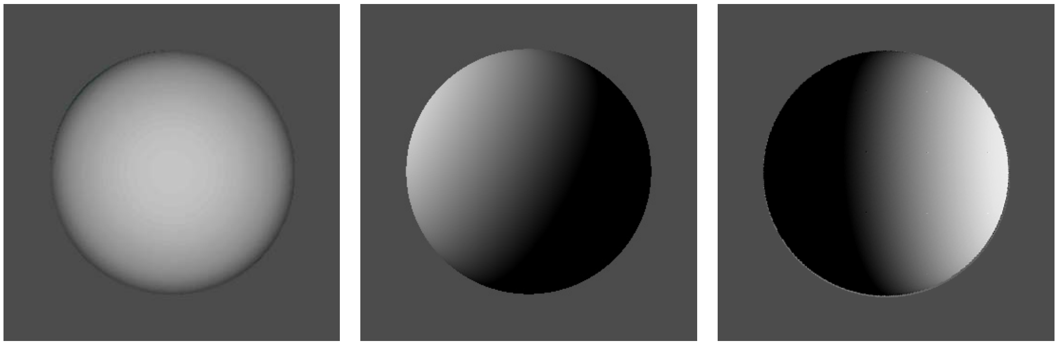

Figure 2. Sphere under various illumination schemes

And to attain any of these, we would have to use some scheme of illumination other than "uniform, omnidirectional".

Now, to get a little ahead of the story, let's imagine three scenarios in which the photographer use one of the three lighting plans represented in these images. Now for each one, what would be the "ideal" exposure result? If we knew that, then we could think in terms of some metering technique that would lead us to the photographic exposure (combination of aperture and shutter speed) that would give us that exposure result.

Well, our objective can't be the tidy one expressed under "B", above. That objective would be attained with an image that looks like that in figure 1, and we decided that wouldn't be very useful.

And in fact, there is no "tidy, automatic" way to conclude what exposure result would be "ideal" for one of our sphere photos—that would be a matter of the artistic judgment of the photographer.

Hold that thought.

1.6 Cinema/portrait lighting technique

Typically in close-up cinematography and in portrait photography, as with the sphere in our example, we rarely want an image resulting from the use of uniform omnidirectional lighting. Rather, we will generally want to use a more sophisticated lighting technique that will "sculpt" the features of the subject.

Very often, in this situation, the subject of the shot is illuminated by two light sources, under the so-called "key-fill" technique. The *key light* is typically directed to the subject from the side. It serves to create shadows that "sculpt" the face. The *fill light* is typically directed to the subject from near the camera position. Its job can be looked at as "diluting" the shadowing from the key light to retain just the degree of sculpting desired by the cinematographer or photographer.

The result of course departs dramatically from the premise of Objective B, which is that in the final image, the illuminance of each element of the image is proportional to the reflectance of that element. With the lighting technique I mention above, the left cheek of our subject may have a high reflectance, but we intentionally light the subject so the left cheek is "in shadow", and is thus given a low luminance in the image.

What should be our objective for the "distribution" of photometric exposure over the image in such a case? As with our little exercise in photographing the stone sphere, there is no simple answer. (This is a dilemma we will encounter repeatedly in this article.)

Thus clearly we cannot devise an single exposure metering technique that will, on a theoretical basis, deliver the "ideal result", since we can't even define what that is.

1.7 Duplex metering

Faced with this conundrum, over the years cinematographers (and portrait photographers) found, empirically, that an exposure result in the image that they considered "desirable" could usually be attained by what came to be called the "duplex" technique of incident light exposure metering.

Here, an incident light exposure meter is used to take separate readings while its receptor faced the two principal light sources. The average of the two meter readings is used as the input to the calculator to develop the photographic exposure recommendation.

2. The Norwood principle

2.1 Donald W. Norwood

Donald W. Norwood had been a photographer in the US Army Air Corps in the period after World War I, and had in fact during that service devised some improvements in photographic processing. After he left the service, it seems as if his attention was directed to cinematography (although it does not seem that he actually practiced that craft professionally).

2.2 Incident light exposure metering in the mid-1930s

In the mid-1930s, incident light metering had become common in cinematography, as is was seen as leading to the "most consistent" results over a range of scenes. Typically, the "duplex" technique (see section 1.7) was used, requiring two or more measurements to be used to prepare for each shot, a burdensome matter where "time was money".

2.3 Norwood's vision

Don Norwood, pondering this inconvenient situation, had a vision of a scheme by which a single measurement would directly give an "appropriate" exposure recommendation over a range of key-fill lighting setups.

The scheme revolved around a measuring instrument in which the photosensitive element had the form of a hemisphere (as contrasted to the "flat" photosensitive element typically used theretofore in incident light exposure meters). He later realized that the same behavior could be attained at less manufacturing cost by using a translucent hemispherical "light collector" (a "dome") mounted over a conventional flat photosensitive element.

Norwood received a patent on this system in 1940.

2.4 A great success, to this day

Work done with prototypes of exposure meters following Norwood's principle seemingly gave highly satisfactory results, and soon commercial meters (made under Norwood's patent) were "all the rage" among cinematographers.

In figure 3, we see the first "Norwood Director" exposure meter made, under Norwood's patent, starting in 1947. (The design work had started in 1941, but the company became devoted to the war effort, which delayed the completion and release of this product.) This product came to be called, by meter aficionados, the "Norwood Director Model A"; that model designation was never used by the manufacturer.

Figure 3. Norwood Director exposure meter ("Model A")

We can hardly miss the "dome" (actually about 1.5 inch in diameter).

A short while later, a second manufacturer was also licensed under Norwood's patent, and developed the meter we see in Figure 2 (introduced in 1948). It also carried the name "Norwood Director", and was identified as "Model B" out of respect for its progenitor, often called the "Model A" (even though that was made by a different company and that designation was never official).

Figure 4. Norwood Director Model B exposure meter
From the collection of Carla and Doug Kerr

Photo by Douglas A. Kerr

This widespread acceptance of Norwood's principle has continued to this day. Almost every "serious" incident light photographic exposure meter made today follows Norwood's principle, which we can easily recognize from the prominent white domes they all sport. We see a typical modern such meter, this one digital, in figure 5.

Figure 5. Sekonic Model L-408 exposure meter
Photo by Kyu Hachi

In fact one of the many models made by Sekonic today—in the vein of a "classic"—is almost identical to the meter seen in figure 2, which was designed at least 60 years earlier. We see it in figure 6.

Figure 6. Sekonic Model L-398A exposure meter
From the collection of Carla and Doug Kerr

Photo by Douglas A. Kerr

3. BUT HOW DO IT KNOW?

3.1 A photometric model?

Understandably, upon the emergence of the Norwood-type meter, engineers and scientists interested in this area were anxious to develop a model, based on known principles of photography and photometry, that would explain how and why a "Norwood" meter could consistently yield photographic exposure recommendations that were felt to be "highly appropriate" over a range of lighting situations.

This quest for insight was greatly burdened by the fact that we had no objective "metric" by which we could judge the "appropriateness" of the exposure result in an image, and thus objectively score how "appropriate" was the recommendation of the exposure meter.

3.2 No real help from Norwood

Those seeking to develop such a model got little help from Norwood, who did not offer for many years any technically-meaningful "rationale" for the working of his system. (He later suggested that this was because the protection of his principle by patents was not yet complete.)

In fact, the only early insight into the rationale comes from this introductory passage in Norton's definitive patent on his system, issued in September, 1940:

> "One of the particular objects of the invention is to provide an exposure meter which is substantially uniformly responsive to

> light incident upon the photographic subject from practically all directions which would result in the reflection of light to the camera or other photographic register."

Now this sounds nice, but is not too helpful in understanding why this should lead to the useful performance we associate with Norwood-principle meters. And in fact it is not even true. The theoretical response of a hemispherical "collector" is not uniform from all directions of interest, but rather declines with increasing angle of arrival of the light, following a mathematical curve known as the *cardioid*. The derivation of this is given in Appendix B.

But it certainly possible that early in his work, Norton assumed that the response of a hemispherical collector meter would be uniform from all directions (at least all directions of interest). Later, in a seminal paper (discussed in Appendix A), we find him expressing a different (but still incorrect) view of the expected response of a hemispherical collector.

3.3 Another outlook

Not too long after the introduction of the Norwood system meter, Norwood pointed out that the hemispherical light collector was a approximate proxy for the human head—that is, the part of it that can be seen from the camera. If we follow the photometric trail, that means that the meter reading would indicate the average[4] illuminance on the part of the subject visible to the camera.

But tidy as this sounds, it still leaves us with the question, "Why would a photographic exposure based on the average illuminance on the part of the subject visible to the camera lead to an 'appropriate' exposure result (whatever that is) for a range of lighting setups?"

3.4 Something more "scientific"

In 1950, Don Norwood published a paper before the Society of Motion Picture and Television Engineers ("Light Measurement for Exposure Control", *J SMPTE 1950, 54:585-602*) that gave a helpful outlook into that mystery, not through an abstract mathematical model but rather through analysis of empirical observation in a test program. The presentation is riddled with (to me disappointing) lapses of rigor (perhaps even of candor), but fortunately these do not invalidate the practical conclusion.

I discuss (and critique) this paper in some detail in Appendix A.

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[4] Average by surface area, to be precise.

Briefly, Norwood found that, in a key-fill lighting setup, for each of several angular positions of the key light, there was a certain photographic exposure (greater than the exposure used for a comparison shot with the key light at the camera) which produced an image which observers adjudged to be "comparable in appearance" to the comparison shot (whatever that might be).

Norwood then went on through several stages to demonstrate that the response of a hemispherical-collector meter *vs*. the angle of the light hitting it[5] would be such that the meter would give an exposure indication that would exactly be the exposure which the subjective tests had shown was needed to produce a consistent "visual appearance" of the image.

Sadly, the development of this conclusion is riddled with the kind of gaffes that would have caused the paper to be sent back by any credible peer review board. I describe these in Appendix A.

But the good news is, despite the lack of forensic credibility created by these gaffes, the ensuing numerical discrepancies are not large at all, and overall this paper still demonstrates that the readings of a Norwood system meter are a good guide to photographic exposure over a range of situations of key-fill lighting.

4. Comparison with the "duplex" technique

We started by pointing out that, prior to the emergence of the "Norwood" metering concept, the "duplex" technique was often used to develop a photographic exposure recommendation in such cases as key-fill lighting. Seemingly, there was general satisfaction with this technique, other than that is was time-consuming.

The Norwood system allowed the photographic exposure recommendation to be determined with a single measurement, clearly an improvement in efficiency.

It is then interesting to ask, "For a given key-fill lighting setup, would the duplex technique and a Norton system meter theoretically yield approximately the same photographic exposure recommendation?"

Yes. Table 1 gives the results of a simulation done here, comparing the photographic exposure recommendations developed with the duplex metering technique (assuming an exposure meter with *cosine* directivity) and an ideal "Norwood" meter (with *cardioid* directivity). The assumed key:fill ratio is 8:1 (as in Norwood's paper).

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[5] This is technically referred to as the *directivity pattern* of the meter.

The relative exposure recommendations are shown first as the actual relative numerical value, followed (in italics) by the equivalent in stops.

Key light angle	Relative exposure recommendation (duplex)	Relative exposure recommendation (Norwood)
0°	1.00/*0.00**	1.00/*0.00**
45°	1.17/+*0.22*	1.15/+*0.20*
90°	2.00/+*1.00*	1.80/+*0.85*

* By definition

Table 1.

The simulation was not done for the 135° case, where both techniques are in fact "dicey". (This is where the key light is a bit behind the subject, which raises many further complications.)

As you can see, the agreement between the two techniques is quite good.

5. Conclusion

Norwood's introduction of the hemispherical collector exposure meter, almost certainly at first based more on intuition than scientific principle, made a gigantic and long-lasting improvement in the art of incident light exposure metering, especially in the cinematographic arena. It seems quite fitting that, in April, 1969, he was given an Academy Award for this work.

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Appendix A
Norwood's 1950 SMPTE paper

A.1 Introduction

In 1950, Donald W. Norwood published a paper before the Society of Motion Picture and Television Engineers (SMPTE) ("Light Measurement for Exposure Control", *J SMPTE 1950, 54:585-602*) that gave a helpful outlook into the way in which the hemispherical-collector exposure meter gives appropriate photographic exposure recommendations over a range of lighting situations.

It did this not through an abstract mathematical model but rather through analysis of empirical observations in a test program.

The presentation is riddled with (to me disappointing) lapses of rigor (perhaps even of candor), but fortunately these do not invalidate the practical conclusion.

A.2 The test program

The test program pertains solely to photography of the human face using key-fill lighting technique (certainly a preoccupation of cinematographers as well as portrait photographers, then and now).

In the tests, for each of several human subjects, shots were taken with the key light (of consistent "potency") at angles (from the camera) of 0° ("head on"), 45°, 90°, and 135°. All shots included the use of fill light from approximately the camera position (at a key-to-fill ratio of 8:1).

The photographic exposure used for the "head-on" shot was based on measurement of the composite illuminance, with that lighting setup, on a camera-facing plane at the subject, using the generally-accepted incident light exposure metering equation.[6]

For each other key light angle, several shots were taken with various amounts of photographic exposure greater than that used for the head-on shot, in 1/2-stop increments.

Then, for each series of shots at a certain key light side angle, a group of observers were asked which shot "matched in appearance" the head-on lit shot of the same subject.

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[6] Which would approximately fulfill "objective B" as expressed in the body of this article.

Here we run into a problem. Clearly none of the side-lit shots would “match in appearance” the head-on shot, as the “sculpting” of the face would be quite different.

We have no idea what the instructions to the observers actually were in this regard. Perhaps the observers were actually asked which of the side-lit shots “looked to have the same overall exposure result as the head-on shot”, or perhaps, even better, “looked to have an overall exposure result that was ‘equally as appropriate’ as that of the head-on shot”, or maybe even “equally nice”.

A.3 Analysis and interpretation

In any case, statistical analysis of the response data led Norwood to the conclusion that, over the range of subjects used, the photographic exposure required in a side-lit shot to get “visual parity” (whatever that was) with the head-on shot was consistently greater than the photographic exposure for the head-on shot by an amount that increases with the angle of the key light. The degree of that needed additional exposure is shown in column 2 of table A1 (in the form stated in the paper):

1. Key light angle	2. Needed additional exposure (stops)	3. Implied relative effective illumination
0°	0*	100%*
45°	½–	75%
90°	1	50%
135°	2	25%

* By definition

Table A1

The entry for 45° presumably means “a little less then 1/2 stop”.

We see that at a 90° position of the key light, that increase in the needed photographic exposure was exactly one stop. Norwood commented that this matched a familiar rule of thumb of photographers at the time (and still used) for that situation. This was thus a reasonable “credibility check” point for the results of the tests.

He then considers the inverse of this needed increase in photographic exposure as a declining “effective value" of the illumination offered by the combinations of the two light sources, which he expressed as a percentage of the effective value for the head-on setup. We see those values in column 3 of Table A1. They follow a precisely declining linear relationship with angle. (How lovely.)

We will move to Figure A2 for the next part of the story. For continuity, we show the implied relative effective illumination (from column 3 of Figure A1) in column 2.

Norwood then says, in effect, that, if the relative reading of the meter followed that relationship (as shown in column 3 of Figure A2), then the exposure recommendation from the meter would be exactly that needed to produce the "visually equivalent" exposure result.

1. Angle	2. Implied relative effective illumination	3. Needed relative meter reading	4. Needed meter directivity (per Norwood)	6. Expected directivity of hemispherical receptor meter (Per Norwood)
0°	100%*	100%*	1.00*	1.00*
45°	75%	75%	0.75	0.75
90°	50%	50%	0.50	0.50
135°	25%	25%	0.25	0.25

* By definition

Table A2

He then goes on to say that for this to happen, the directivity of the meter would have to follow that same relationship, as shown in column 4 of Figure 2.

Norwood then tells us that in fact the expected directivity of a hemispherical collector meter would be that very same linear relationship (as shown in column 6 of Table 2). (The peculiar numbering of that column is for compatibility with another table to be seen shortly.)

Thus, said Norton, from the precise equivalence of columns 4 and 6, we can see why the behavior of his exposure metering system gives the consistently good result for which it had by then become famous.

A.4 But wait a minute

If we examine this train of thought, we encounter several disturbing matters.

The first is in the needed values of excess exposure. Except for the 45° key light angle, these are all integral values in stops. (We note that there is no mention in the paper of this value being "rounded to the nearest 1/2 stop" or any such.) The relationship (if we choose to make a certain arbitrary refinement of the "1/2–" value), is perfectly inversely linear with angle.

And in fact, the inverse quantity (the relative effective luminance, as seen in column 3 of Table A1) is precisely linear with angle. Considering that the source of this is a highly subjective set of observations, and that the underlying physical model involves a number of trigonometric relationships, this perfectly linear result is nothing short of astounding.

And the situation at 45° is especially interesting. Starting with the casually-stated value of needed excess exposure, "1/2–", and taking its "inverse", Norwood gets exactly 75%. (As a matter of interest, the value of excess exposure needed to lead to a relative effective illuminance of 75% would be about 0.42 stop. I guess that qualifies as "a little less than 1/2 stop".)

Next we look at the matter of the required directivity of the meter. We will move at this point to Figure A3, which is an expansion of Figure A2.

Norwood tells us that if the directivity of the meter matched the relationship seen in column 3, then the meter reading would follow the relationship seen in column 3, and thus the exposure recommendation of the meter would exactly match what is needed here. Such a meter directivity is shown in column 4.

1. Angle	2. Implied relative effective illumination	3. Needed relative meter reading	4. Needed meter directivity (per Norwood)	5. Needed meter directivity (actual)	6. Expected directivity of hemispherical receptor meter (Per Norwood)	7. Expected directivity of hemispherical receptor meter (actual)
0°	100%*	100%*	1.00*	1.00*	1.00*	1.00*
45°	75%	75%	0.75	0.714	0.75	0.853
90°	50%	50%	0.50	0.428	0.50	0.500
135°	25%	25%	0.25	0.142	0.25	0.146

* By definition

Table A3

But that's not so. For any fill light position, the meter receives not only light from the fill light (to which its response is based on its directivity at that angle) but also light from the fill light, (to which its response is based on its directivity at angle 0°. The meter reading is the sum of those two responses. Especially at the larger angles, where the meter directivity is fairly low, the contribution of the fill light to the meter reading is not trivial.

If we do the algebra, we find that what is needed so that the meter's reading will follow column 3 (and thus lead to the needed exposure) is for the meter's directivity to be as seen in column 5 of Table 3.

Next, Norwood tells us that the expected directivity of a hemispherical-collector meter is as seen in column 4 of Table 3. [7] But that's not true. The expected directivity of a hemispherical-collector meter is as shown in column 6 of Table 3. Plotted as a polar curve, this directivity pattern would be a *cardioid*.[8] The derivation of this is given in Appendix B.

Norwood points out that the "exact" equivalence of columns 4 and 6 of Table 3 shows that a ideal hemispherical-collector meter would give an exposure recommendation corresponding exactly to what was seen, from the tests, to be needed for consistently "visually-equivalent" exposure results over the range of key light positions. But columns 4 and 6 are not valid; columns 5 and 7 are.

A.5 Grading the paper

The lapses from rigor in the trail of Norwood's "derivation" are disturbing, and would certainly have earned this paper a "thumbs down" had it been subject to peer review. And to the cynical forensic engineer (who, me?), they raise serious questions as to whether this story with its amazingly-tidy result was formulated in fully good faith by the author.

Did Norwood by any chance "work backwards" from a perfect result, taking artistic liberties with the mathematical relationships actually involved on the way? Or was he just careless with his work? I leave it to the individual reader to contemplate that.

A.6 The good news

That indictment aside, the actual numerical errors in Norwood's presentation resulting from his lapses in "logic" are of modest size. The greatest discrepancy, at 45°, corresponds to only about 0.25 stop in photographic terms.

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[7] Plotted as a polar curve, this directivity pattern would be an *Archimedean spiral*.

[8] Plotted as a polar curve, this directivity pattern would be a *cardioid*.

And in any case, we are speaking of a situation in which there is no "exactly correct" result.

Thus it seems to me that Norwood's story, even if we replace his numbers with the valid ones, quite credibly demonstrates that a Norwood-principle exposure meter can be expected to yield photographic exposure recommendations that, over a range of key-fill lighting situations, lead to "appropriate" image results.

What about other lighting situations, including general outdoor scenes? I have no information on studies done of such situations.

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Appendix B
Derivation of the cardioid directivity of the hemispherical dome

B.1 The cardioid curve

The expression for a *cardioid* curve, in polar coordinate form, normalized to a maximum value of 1.0, is:

$$R = \frac{1 + \cos\Theta}{2} \qquad (1)$$

B.2 The directivity response of the hemispherical receptor

We assume that the "directivity pattern" of a hemispherical incident light metering receptor (including as implemented with a flat receptor covered by a translucent hemispherical dome) is proportional to the projected area of the dome as seen from the angle of interest. (That area determines how much luminous flux the dome will capture from a beam of any given luminous flux density.)

B.3 The projected area of a hemisphere from various angles of observation

We will work from figure 7.

Panel a—"head on" observation

In panel a of the figure, we see the projected area of the dome as we would see it from a point on its axis. Here Θ, the angle of observation, is 0. The cosine of Θ is 1.0. We use A to represent the projected area as seen from $\Theta = 0$ (that is, as seen in this panel). A will mean that very same area in future panels.

In order to set the stage for our future work, I divide the projected area into two equal portions by a vertical dotted line. The area of each portion is A/2.

Note that in this case, the boundary of the projected area is in fact identical to the "rim" of the hemisphere as seen from our vantage point. Accordingly, in this view, the area of each half of the projected area of the hemisphere is of half the area of the circle defined by the rim of the hemisphere. This is in turn determined by the radius of the hemisphere, *R*.

Panel b— observation from an angle of 60°

In panel b, we have moved our vantage point to the right by 60°, so that Θ, the angle of view of the hemisphere, is 60°. Cos Θ is 0.5.

The left boundary of the projected area is no longer the left half of the rim of the hemisphere, which has moved "around back"–just the leftmost "limb" of the hemisphere. But the right boundary of the projected area is still the right half of the rim, which has now moved "around to the front". That half of the rim is a semi-circle, but, since we see it from an angle to its plane, we see it foreshortened as a semi-ellipse.

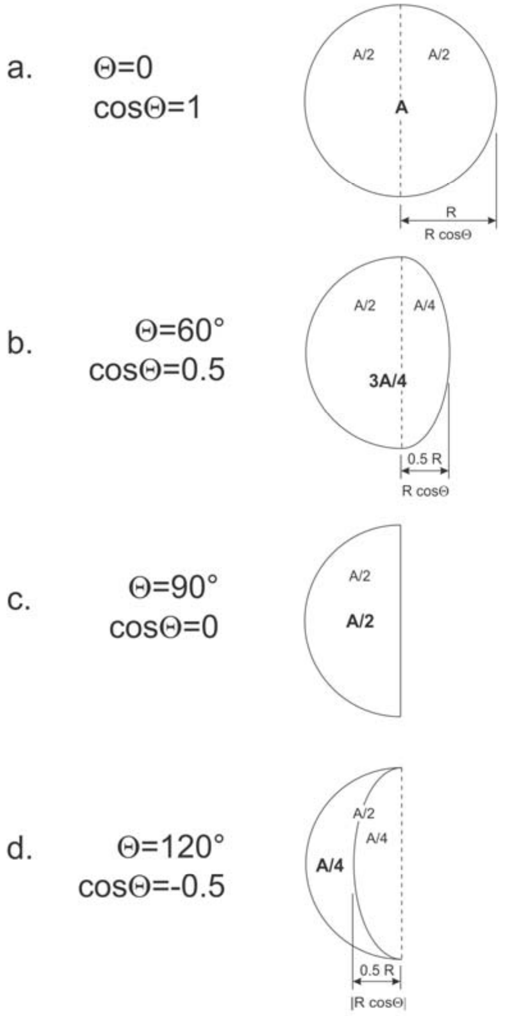

Figure 7. Projected area of the hemisphere

As a result of this foreshortening, the horizontal semidiameter of that projected ellipse is R cos Θ. Said another way, the width of that semicircular area is reduced by the factor cos Θ. And thus the area itself is reduced by the factor cos Θ.

Therefore the area embraced by that right-hand semi-ellipse is (A/2) cos Θ, or A/4. Thus the entire projected area of the hemisphere, the sum of the two sections, is 3A/4. (That is shown in bold.)

Panel c— observation from an angle of 60°

In panel c our view is from 90° to the right. We note that for $\Theta = 90^\circ$, $\cos\Theta = 0$.

Now the "near half" of the rim of the hemisphere is seen "head on", and collapses to a vertical line; we do not see it.

Accordingly, the projected area of the hemisphere is just A/2.

Panel d— observation from an angle of 60°

In panel d, our view is from 120° to the right. We note that for $\Theta = 120^\circ$, $\cos\Theta = -0.5$.

As in panel b, the right boundary of the projected area is the projection to us of the "near" half of the rim of the hemisphere, now "flipped" left of the dotted centerline. Again, its horizontal semidiameter is $R \cos\Theta$ (but, to be rigorous, since $\cos\Theta$ is negative, we must state that (positive) distance as the absolute value of $R \cos\Theta$.

Thus, the total projected area, A', is the "left portion" area, A/2, diminished by the area in the semiellipse, $(A/2) \bullet |\cos Q|$ (which comes to A/4), a net area of A/4.

Summary

We see that in every case, geometrically, the net projected area of the hemisphere is an area of A/2 to which we add an area of $(A/2) \cos\Theta$ (noting that for $Q > 90^\circ$, $\cos\Theta$ is negative, so that area then would actually be subtracted).

Algebraically, then, the projected area of the hemisphere from a point at angle Θ is consistently given by:

$$A' = \frac{A}{2} + \frac{\cos\Theta\, A}{2} \qquad (2)$$

or

$$A' = \frac{1 + \cos\Theta}{2} A \qquad (3)$$

Thus the relative sensitivity of the receptor, s, which we have assumed is proportional to the projected area of the hemisphere from the angle of interest, is:

$$s = \frac{1 + \cos\Theta}{2} \qquad (4)$$

which is identical with the expression, in polar coordinates, for a cardioid curve:

$$R = \frac{1 + \cos \Theta}{2} \qquad [1]$$

Quod erat demonstrandum.

B.4 A departure

Almost certainly, in the usual implementation, for angles of incidence beyond 90° there would be some obscuration of the dome by the meter housing. Thus we might expect for such greater angles the actual response would decline faster than as predicted by the cardioid curve.

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